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ABSTRACT

After a brief summary of their development, the mathematics objectives of the National Assessment of Educational Progress are classified under three dimensions: (1) The Use of Mathematics dimension has three levels: social, technical, and academic. Within each level, a hierarchy of subject matter and skills from easy (90 percent correct) to very hard (10 percent correct) is projected; (2) The Content domain includes all mathematics currently taught in the elementary and secondary schools of the nation, up to but not including the calculus. This content is listed under 17 main headings; and (3) The Objectives or Abilities dimension has six levels: recall, manipulation, understanding concepts, solving problems, open-ended applications, and appreciation of mathematics. The general nature of the tasks within each level is described, and the specific topics for each age (9, 13, 17, and adult) are listed; but no illustrative test items are included. (MM)

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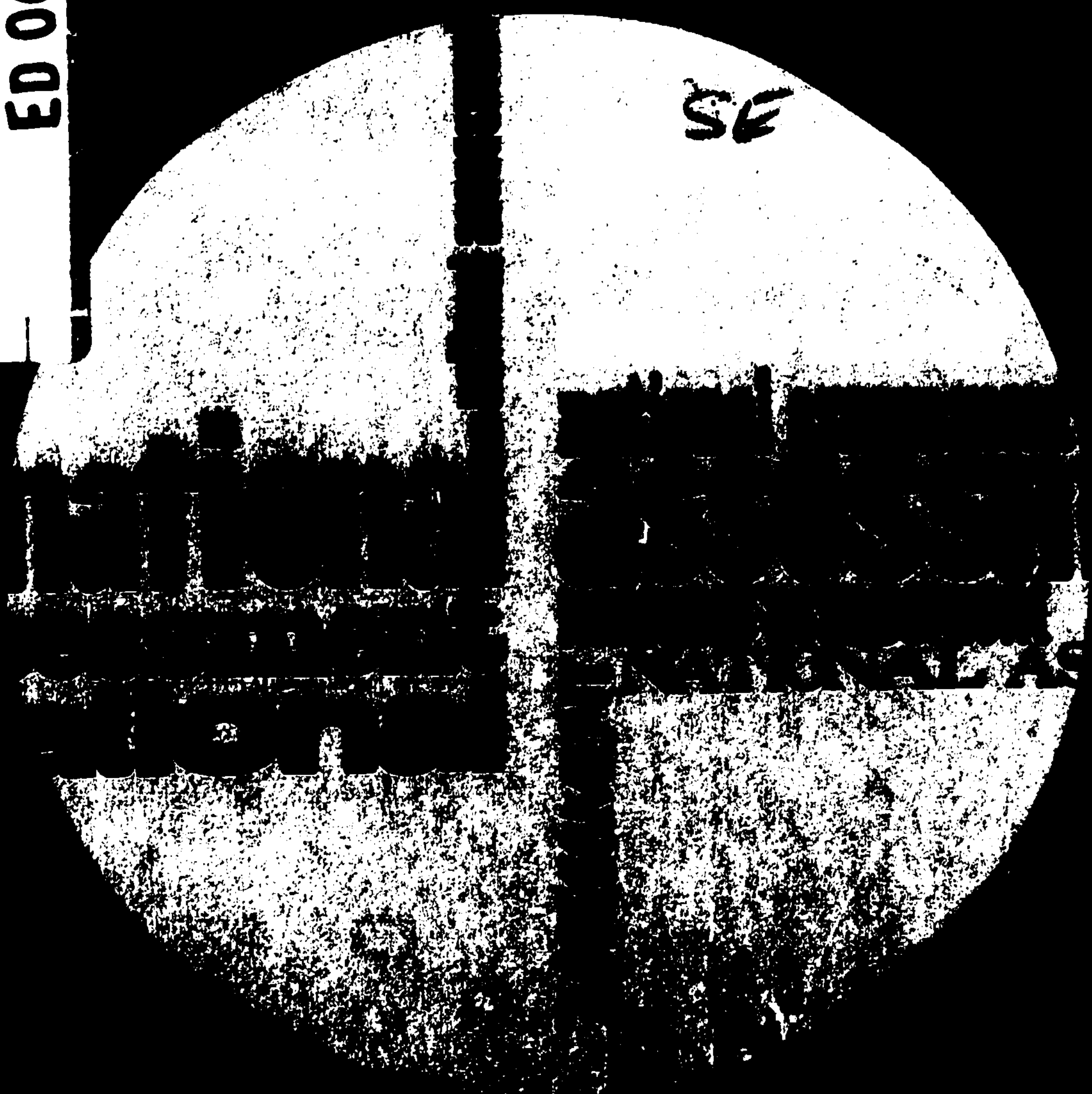

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National Assessment of Educational Progress

Mathematics Objectives

National Assessment of Educational Progress

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PREFACE

After more than four years of effort in developing its plan and instruments, the National Assessment of Educational Progress began actual assessment in the spring of 1969 with the administration of exercises to a random sample of 17-year-old students in schools throughout the United States.

The educational objectives from which exercises were developed in mathematics are published here, together with an introduction to the project. The procedures followed by National Assessment staff and its contractors in developing the mathematics objectives are described in the second chapter, followed by the objectives themselves.

Although names of experts, lay panel chairmen, and some of the educational organizations deeply involved in developing the objectives appear in the appendices of this booklet, it is impossible to give proper recognition to all who contributed to the development of the objectives and their publication. However, we want to particularly acknowledge the contributions of William A. Mehrens, Jack C. Merwin, Dale C. Burklund, Mrs. Frances S. Berdie, Dale I. Foreman, Edward D. Roeber, and Mrs. Peggy A. Bagby to the preparation and publication of the objectives in their final form.

Eleanor L. Norris
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Editors

National Assessment welcomes your comments on the objectives in this brochure or any other phase of National Assessment activity. We would also like to encourage your suggestions for new or revised objectives. Comments should be addressed to:

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. . . . . *the editors*

TABLE OF CONTENTS

Chapter I

INTRODUCTION

The National Assessment is designed to furnish information to all those interested in American education regarding the educational achievements of our children, youth and young adults, indicating both the progress we are making and the problems we face. This kind of information is necessary if intelligent decisions are to be made regarding the allocation of resources for educational purposes.

In the summer of 1963 the idea of developing an educational census of this sort was proposed in a meeting of laymen and professional educators concerned with the strengthening of American education. The idea was discussed further in two conferences held in the winter of 1963-64, and a rough plan emerged. The Carnegie Corporation of New York, a private foundation, granted the funds to get started and appointed the Exploratory Committee on Assessing the Progress of Education (ECAPE). The Committee's assignment was to confer at greater length with teachers, administrators, school board members and other laymen deeply interested in education to get advice on ways in which such a project could be designed and conducted to be constructively helpful to the schools and to avoid possible injuries. The Committee was also charged with the responsibility for getting assessment instruments constructed and tried out and for developing a detailed plan for the conduct of the assessment. These tasks required four years to complete. On July 1, 1968 the Exploratory Committee issued its final report and turned over the assessment instruments and the plan that had been developed to the Committee on Assessing the Progress of Education (CAPE), which is responsible for the national assessment now under way.

In the early conferences, teachers, administrators and laymen all emphasized the need to assess the progress of children and youth in the several fields of instruction, not limiting the appraisal to the 3 R's alone. Hence, the first assessment includes ten areas: reading,

writing (written expression), science, mathematics, social studies, citizenship, vocational education (career and occupational development), literature, art, and music. Other areas will be included in the second round. The funds available were not sufficient to develop assessment instruments in all fields of American education. The ten chosen for the first round are quite varied and will furnish information about a considerable breadth of educational achievements.

Because the purpose of the assessment is to provide helpful information about the progress of education that can be understood and accepted by laymen as well as professional educators, some new procedures were followed in constructing the assessment instruments that are not commonly employed in test building.

These procedures are perhaps most evident and important in the formulation of the educational objectives which govern the direction of the assessment in a given subject matter area. Objectives define a set of goals which are agreed upon as desirable directions in the education of children. For National Assessment, goals must be acceptable to three important groups of people. First, they must be considered important by scholars in the discipline of a given subject area. Scientists, for example, should generally agree that the science objectives are worthwhile. Second, objectives should be acceptable to most educators and be considered desirable teaching goals in most schools. Finally, and perhaps most uniquely, National Assessment objectives must be considered desirable by thoughtful lay citizens. Parents and others interested in education should agree that an objective is important for youth of the country to know and that it is of value in modern life.

This careful attention to the identification of objectives should help to minimize the criticism frequently encountered with current tests in which some item is attacked by the scholar as representing shoddy scholarship, or criticized by school people as something not in the curriculum, or challenged by laymen as being unimportant or technical trivia.

National Assessment objectives must also be a clear guide to the actual development of assessment exercises. Thus, most assessment objectives are stated in such a way that an observable behavior is described. For example, one citizenship objective for 17-year-olds is that the individual will recognize instances of the proper exercise or denial of constitutional rights and liberties, including the due process of law. Translated into exercise form, this objective could be presented as an account of press censorship or

police interference with a peaceful public protest. Ideally, then, the individual completing the exercise would correctly recognize these examples as denials of constitutional rights. It should be noted, however, that exercises are not intended to describe standards which all children are or should be achieving; rather, they are offered simply as a means to estimate what proportion of our population exhibit the generally desirable behaviors implicit in the objectives.

The responsibility for bringing together scholars, teachers, and curriculum specialists to formulate statements of objectives and to construct prototype exercises was undertaken through contracts by four organizations experienced in test construction, each responsible for one or more subject areas. In several areas the formulation of objectives was particularly difficult because of the breadth and variety of emphases in these fields. Hence, two contractors were employed to work on each of these areas, independently, in the hope that this would furnish alternative objectives from which panels composed of lay persons could choose.

This brief description of the process employed in identifying objectives for the first assessment should furnish a background for examining the sections that follow in which the objectives and prototype exercises are presented. The instruments actually used in the assessment provide samples of exercises appropriate for the four age groups--9, 13, 17, and young adults from 26-35–whose achievements are appraised, and for the wide range of achievement at each age.

Chapter II

PROCEDURES FOR DEVELOPING MATHEMATICS OBJECTIVES

The development of Mathematics objectives was a complicated process involving two contractors, a math education consultant, advisory panels composed of mathematicians and math educators, and a panel of interested laymen who reviewed the objectives for their suitability.

The Educational Testing Service (ETS) and The Psychological Corporation were both awarded contracts in the spring of 1965 to develop Mathematics objectives which were to be submitted to the ECAPE staff in the fall of that year. Both contractors followed the same general pattern in developing their objectives. Each convened, along with its staff, its own panel of mathematicians and math education specialists to develop objectives. The contractors and their respective panels were asked to keep in mind the criteria established by ECAPE listed in the introduction as well as several other important considerations:

1. National Assessment would be directed at four age groups–9, 13, 17, and young adults. To be truly national in character, parochial, private, and public schools all should be involved.
2. The objectives and the instruments developed from them should cover a wide range of difficulty levels. This meant including tasks which almost all of the population at a given age level could complete, tasks which about half could complete, and tasks which only the most knowledgeable and highly skilled could complete.

ETS panel members were also given a memorandum on the levels of cognitive behavior or abilities to be considered in the

assessment and an Item File Classifications Scheme developed for the National Longitudinal Study of Mathematical Abilities.

Both sets of objectives, when completed, were turned over to panels of interested lay citizens who were to decide which set should be used in the assessment. The general feeling among lay people was that objectives developed by ETS tended to emphasize "practical" mathematics while those developed by The Psychological Corporation seemed more oriented toward "academic" or scholastic mathematics. Unfortunately, panel members were evenly divided in their preference for the two sets of objectives. Panel members preferring the ETS set considered it a better basis on which to make an assessment because these objectives seemed to stress both skills and the importance of the thinking process. Panelists preferring The Psychological Corporation objectives considered them to be a more dynamic approach to mathematics and a more readily operational set for actual assessment purposes. The principal objection to the ETS set was that some panel members thought these objectives should consider computers and computer programming. Panelists also had several criticisms of The Psychological Corporation objectives, feeling (a) that adults probably should not be assessed in the area of set theory since this type of instruction is a relatively recent innovation and (b) that there was not enough content for the top 10 percent of students, notable in the omission of college and professional math, calculus, vector analysis, and computer programming from the objectives.

The panels' indecision made it necessary for the ECAPE staff to select the objectives to be used for further development. The indecision was resolved by asking The Psychological Corporation to continue the development of their objectives and exercises in order to better equalize the work load among the several contractors[1] involved in developing other assessment subject areas. However, it is still necessary to consider in more detail the development of objectives by both organizations, since many ETS suggestions were ultimately incorporated into the final set of objectives used in the assessment.

By 1966, The Psychological Corporation, in consultation with its panel of experts, had developed a statement of objectives in the form of a two-way grid for each age level considered in the assessment. Along one axis of each grid were nine behavioral objectives, ranging from simple ability to recall definitions, notations, operations, and concepts to complex tasks, such as analyzing problems and determining which mathematical opera-

[1] ETS was awarded contracts in six subject areas.

tions were needed to solve the problem. The other grid axis specified math curriculum content areas to be considered for each age level. Each grid provided a convenient format for indicating expected behaviors for a given math content area at each age level.

The beginning efforts by The Educational Testing Service and its panel of mathematics experts resulted in a somewhat different organization of mathematics, consisting of three basic steps. They first considered mathematics from the standpoint of its possible uses. Secondly, mathematics was discussed in terms of the specific operations, skills, or content areas to be covered. Finally, the committee considered formulation of the objectives themselves, basing the construction upon various uses for math and the mathematical operations necessary to fulfill a given use or task.

The ETS panel's greatest contribution to the final set of objectives came from two sources. Six behavioral objectives, constituting the major subdivisions of the outline presented in the next chapter, were developed by ETS and were eventually adopted by the NAEP staff as the basic organizational scheme for the Mathematics objectives. The second source was an ETS document detailing three areas of mathematics use. The first area, social mathematics, included all mathematics important to personal living and citizenship in the society, covering such skills as reading and the use of symbols, basic arithmetic, and simple measurement. A second area was concerned with technical mathematics and included calculations that would be necessary for various skilled jobs and professions beyond the simple base of social mathematics. The final area, academic mathematics, considered math as a formal system to be studied in and of itself. Within each of these classifications, the panel defined a hierarchy of subject matter and skills, ranging from simple to sophisticated and from easy to hard.

By 1968, The Psychological Corporation had completed its task of developing Mathematics objectives and exercises and turned their work over to National Assessment staff for review, possible revision, and ultimate use in the field. Also available to the NAEP staff was the initial work done by ETS and the advice of independent consultants.

Drawing upon the developmental work of these two organizations, the NAEP staff developed a final set of objectives, principally by modifying the objectives grids developed by The Psychological Corporation. The first and most major change was to refine and simplify the grids so that content areas, to the extent possible, were generally consistent across all four age levels considered in the assessment. However, it was not possible to assess certain content areas in some of the age groups. For

example, trigonometry was considered too difficult for the 9-year-old level. This simplified grid was again revised by replacing its nine behavioral objectives with a somewhat similar but more concise set developed by ETS. In addition, the three areas of math usage defined by ETS were also incorporated into the grids. Finally, several content areas minimized or omitted in the objectives grids were included or given increased emphasis.

In the summer of 1968, the revised grids were sent to Emil Berger, a mathematics consultant with the St. Paul Public Schools (Minnesota), for additional revisions and suggestions. By the end of the summer, the objectives were considered to be in final form and ready for use in the first assessment cycle.

Throughout the long development of the objectives, it was necessary to emphasize some areas of mathematics while minimizing others. It was clear to the NAEP staff and contractors that some areas were of greater importance and that administration time for the assessment was not sufficient to do all content areas justice. Areas included and emphasized are clear from the objectives presented in the next chapter. Those minimized for the first assessment cycle were non-Euclidian geometries and business mathematics, though it has been recently decided to give increased emphasis to business mathematics in future assessment cycles. Calculus and transformation of coordinates were not included at all, due to their advanced difficulty and the lack of instruction in schools at this relatively sophisticated level.

National Assessment did not minimize the importance of the individual's interest in and attitudes toward math, considering it important that attention be given to such matters as one's enjoyment of, willingness to use, and active participation in mathematics.

As described earlier in this chapter, thoughtful lay persons have been involved in the selection and reviewing of National Assessment objectives. However, some discussion is warranted concerning how these lay panels were selected in order that they be not only concerned and interested in education, but representative of various sections of the country as well. Lay people interested in education were identified by asking for nominations from various state and national organizations interested in education (see Appendix). From these nominations, persons living in large cities, suburban communities, and rural, small town areas throughout the United States were selected to attend conferences to review the objectives that had been developed. Twelve lay review panels were originally to have been established, representing three different community sizes in each of four major regions of the

country. However, in one region, so few suburban communities existed that only two committees were set up for the region. Each of the remaining 11 committees, chaired by one of the lay panelists, met at a convenient place in the geographic region to discuss the objectives with a member of the ECAPE staff. Each panel reviewed all the objectives developed, providing 11 independent reviews of all 10 assessment subject matter areas. Following the lay panel meetings in each region, the 11 chairmen were brought together for a meeting in New York City in December, 1965, to make their recommendations to National Assessment's Exploratory Committee.

After the objectives for Mathematics (as well as other National Assessment subject areas) were initially developed, they were compared to other statements of objectives in these areas which had appeared in education and mathematics literature during the past 25 years preceding this project. Since the National Assessment objectives were prepared for a specific purpose, their wording and organization were somewhat more uniform than prior statements. However, it was possible to organize these previous statements in terms of their relation to National Assessment objectives. When this procedure was finished, it was clear that National Assessment had not produced "new" objectives in any subject area. Rather, these objectives were restatements and summarizations of objectives which had appeared over the last quarter of a century. This was a desired and expected outcome in that one criterion for National Assessment objectives was that they be central to prevailing teaching efforts of educators.

Objectives presented in the next chapter of this monograph have survived the consideration of both experts and lay people and serve as the basis for exercises which are being presented to four age groups in this first cycle of National Assessment. The task of developing objectives has not ended, however. For as the goals of the educational system evolve and change, so must the objectives used by National Assessment likewise change. This means that there must be continual re-evaluation of the objectives in each National Assessment subject area.

During the summer of 1969, National Assessment began reviewing the objectives for the areas assessed in the spring of 1969: Science, Writing, and Citizenship. Again the assistance of both experts and lay people was requested to determine whether the objectives needed modification. When the first year of assessment in Mathematics is completed, a similar review process will take place. By providing this continuing process of re-evaluation, the National Assessment program hopes that it can attain its

own goal of providing information on the correspondence between what our educational system is attempting to achieve and what, in fact, it is achieving.

Chapter III

MATHEMATICS OBJECTIVES

As a consequence of early attempts to define the objectives of mathematics education and concurrent efforts to determine the scope of subject matter that should be included in an assessment of mathematics, the National Assessment staff with the aid of contractors and consultants, concluded that a three dimensional classification scheme was the most efficient and meaningful method to specify mathematics objectives. The three dimensions consider (1) uses for mathematics, (2) mathematics curriculum content areas, and (3) abilities or behaviors conducive to effective utilization and understanding of mathematics. These dimensions are elaborated below in terms applicable to all age levels presently considered in the National Assessment.

USE OF MATHEMATICS

A three-fold classification of mathematics by use has been defined. Within each class a hierarchy of subject matter and skills from simple to sophisticated and from easy (90% correct) to very hard (10% correct) can be identified.

1. *Social mathematics* includes the mathematics that is needed for personal living and effective citizenship in our society. It includes such things as reading and use of symbols, basic arithmetic, consumer arithmetic, simple measurement and conversion of units of measurement, ratio, estimation, data interpretation, reading of graphs and charts, some intuitive geometry, and general logical thinking. The topics in this category range from the simple knowledge that two nickels have the same value as one dime to the more difficult interpretation of a complex graph in the *Wall Street Journal.*
2. *Technical mathematics* includes the mathematics which is necessary for various skilled jobs and professions beyond that which is needed for personal living and effective citizenship. It ranges in difficulty from the mathematics that the carpenter needs to read scale drawings to the mathematics needed by the engineer, the physicist, the statistician, and the computer specialist. The classification has greater importance for Age 17 and Adult than for ages 9 and 13.
3. *Academic mathematics* is the formally structured mathematics which has assumed increasing importance in the curriculum from kindergarten

through college. The structure, continuity, and cohesion of academic mathematics provide the basis for an understanding of the various isolated mathematical processes. While not obvious in the day-to-day activities of the ordinary citizen, academic mathematics is increasingly important in the general background of the layman. It is also basic for later work in mathematics and many of the other liberal arts; it thus has a place in any national assessment. Academic mathematics includes topics such as sets, field properties, number systems, the nature of proof, etc.

CONTENT

The content domain for the national assessment includes all mathematics currently taught in the elementary and secondary schools of the nation, up to but not including the calculus. The scope of this content for the different age levels is indicated on the accompanying outlines.

A. Number and Numeration Concepts

1. Numeration Systems

   a. Decimal-Place Value
   b. Roman
   c. Other Bases
   d. Modular Arithmetic

2. Number Systems

   a. Whole Numbers
      – Counting
      – Odd and Even Numbers
      – Prime and Composite Numbers
      – Divisibility, Greatest Common Factor, Least Common Multiple
      – Factorials
   b. Integers
   c. Rational Numbers
      – Representation (fraction, decimal, percent)
   d. Real Numbers
      – Real Number Line
      – Irrational Numbers
      – Absolute Value
   e. Complex Numbers

B. Properties of Numbers and Operations

1. Whole Numbers

   a. Order Property
   b. Addition and Multiplication
      – Closure Properties

- Commutative Properties
- Associative Properties
- Distributive Property
- Identity Properties
- Zero Multiplicative Property

c. Subtraction (as inverse of addition)
d. Division (as inverse of multiplication)

2. Rational Numbers

   a. Field Properties
   b. Order Properties
   c. Density Property

3. Real Numbers

   a. Field Properties
   b. Order Properties
   c. Completeness Property

4. Complex Numbers

C. Arithmetic Computation

1. Whole Numbers
2. Rational Numbers (positive and negative)
3. Real Numbers
4. Complex Numbers
5. Ratio, Proportion, and Percent
6. Computation with Approximate Data
7. Rounding Off

D. Sets

1. Properties
2. Operations and Relationships

E. Estimation and Measurement

1. Standard Units

   a. Time
   b. Distance
   c. Area Volume

d. Weight
e. Capacity
f. Temperature
g. Money

2. Conversion Relations

F. Exponents and Logarithms

1. Exponential and Logarithmic Equations
2. Logarithmic Computation
3. Manipulation of Radicals
4. Scientific Notation
5. Using Tables

G. Algebraic Expressions

1. Properties of Expressions (variables, constants, and order of operations)
2. Monomials and Polynomials
3. Rational and Irrational Expressions
4. Manipulation of Expressions
   a. Combining Like Terms
   b. Removing Parentheses
   c. Operations with Expressions
   d. Factoring
5. Evaluating Expressions

H. Equations and Inequalities

1. Linear Equations and Inequalities
   a. Finding solution of equations and inequalities in one variable
   b. Finding solution sets of systems of equations and inequalities in two or more variables
      – Determinants
2. Higher Degree Equations and Inequalities
   a. Finding solution of equations and inequalities in one variable
   b. Finding solution sets of systems in two or more variables

3. Evaluating Formulas

4. Graphic Interpretation of Equations and Inequalities

   a. Graphs of equations and inequalities in rectangular coordinates
   b. Using graphs to find solution sets of equations and inequalities

5. Solving Equations and Inequalities with Absolute Values

I. Functions

1. Definition of a Function—Functional Notation

   a. Domain and Range
   b. Evaluating Functions
   c. Zeros of a Function

2. Linear Functions and Their Graphs

   a. Slope of a Line
   b. y-intercept
   c. Writing Equations of Linear Functions

3. Quadratic Functions and Their Graphs

   a. Writing Equations of Quadratic Functions
   b. Analysis of Graphs of Quadratic Functions

4. Maxima and Minima of Functions

J. Probability and Statistics

1. Basic Probability Concepts

   a. Permutations and Combinations
   b. Outcomes, Samples, Spaces, and Events
   c. Probability of an Event

2. Descriptive Statistics

   a. Measures of Central Tendency
   b. Measures of Dispersion
   c. Measures of Relationship

3. Methods of Representing Data (tables, bar graphs, etc.)

K. Geometry

1. Points, Lines, and Planes

   a. Rays, Segments, and Angles
   b. Sets of Points (locus)

2. Polygons and Polyhedra
3. Circles and Spheres
4. Similarity and Congruence
5. Metric Geometry
    a. Length
       – Pythagorean Theorem
    b. Area
    c. Volume
    d. Angle Measurement
6. Geometric Constructions
7. Coordinate Geometry
    a. Cartesian Coordinates
    b. Polar Coordinates
    c. Distance Formula

L. Trigonometry

1. Trigonometric Functions
    a. The Oriented Angle
    b. Definition of Functions
    c. Graphs of Functions (amplitude and periodicity)
    d. Inverse Functions
    e. Functions of Sums and Differences (half- and double-angle formulas)
2. Relations Among Trigonometric Functions
    a. Trigonometric Functions
    b. Trigonometric Identities
3. Solution of Triangles

M. Mathematical Proof

1. Fundamental Concepts
    a. Basic Terms (definitions, axioms, etc.)
    b. Logical Premises and Rules of Inferences
2. Methods of Proof

N. Logic

O. Miscellaneous Topics

1. Variation and Proportion
2. Sequences and Series
   a. Arithmetic Sequences and Series
   b. Geometric Sequences and Series
   c. Binomial Expansion
3. Vectors

P. Business and Consumer Mathematics

1. Personal and Bank Records
2. Buying
3. Personal Finance
   a. Figuring Take Home Pay
   b. Budgeting Problems (e.g., home and travel expenses)
4. Income from Commissions
5. Borrowing
   a. On a Note
   b. From a Bank
   c. On Collateral
   d. From a Credit Union or Loan Company
   e. Installment Buying
6. Savings, Insurance, and Investments
7. Taxes

Q. Attitude and Interest Items

OBJECTIVES (OR ABILITIES)

The objectives of mathematics education can be described in terms of successive level of developed abilities.

Development of each level of ability can be demonstrated by performance of specific tasks appropriate to each age level. These tasks will include content used in social, technical, or academic settings.

I. *Recall and/or recognition of definitions, facts and symbols.*

A task assessing this ability will require only that the examinee be able to

recognize typical mathematical symbolism, or to recall specific facts. It is the lowest of the levels of cognitive ability in mathematics but is an essential aspect of achievement. Difficulty level in this category will depend more on exposure to the material and on memory than on developed skill.

Age 9

A. Basic addition, subtraction, multiplication, and division facts.

B. Reading and writing Arabic numerals to 100,000.

C. Reading and writing Roman numerals to XII.

D. Reading and writing fractions.

E. Knowledge of vocabulary such as: whole number, equal, add, subtract, multiply, divide, numerator, denominator, proper fraction, mixed numeral, sum, difference, divisor, quotient, remainder, average, place value, round numbers, odd, even, scale drawing, and units of measure.

F. Relationships between inch, foot, yard, mile; between ounces, pounds, tons; between pints, quarts, gallons; between seconds, minutes, hours, days, weeks, months, years; between units of U. S. currency.

G. Recognition of square, rectangle, triangle, circle, right angle, perpendicular lines, parallel lines, intersecting lines.

H. Recognition of inequality and equality symbols.

Age 13

A. Knowledge of the facts of arithmetic, including percent, ratio.

B. Knowledge of the rational and real number systems.

C. Knowledge of geometric figures—similarity, congruence.

D. Knowledge of measurement—direct and indirect.

E. Knowledge of simple algebra.

F. Knowledge of digits required in different numeration systems.

G. Knowledge of terminology: factor, multiple, divisibility, repeating decimal, rational number, integer, prime number, square root, cone, prism, great circle, equilateral, isosceles, vertical angle, commuta-

tive - associative - distributive principles, closure, inverse operation, identity element, latitude, longitude.

H. Knowledge of set notation, set language.

I. Knowledge of sine, cosine, and tangent.

Age 17

A. Knowledge of the facts of arithmetic.

B. Knowledge of elementary algebra.

C. Knowledge of algebraic symbolism, such as:
$<, =, \geq, \leq, >, +, -, x, \div$, AxB, $A \cup B$, $A \cap B$, f(x), log x, exp x.

D. Identification of geometric symbolism, such as:
$\parallel, \perp, \cong, \sim, \angle$ PQR.

E. Knowledge of terms in algebra, such as: variable, linear, and quadratic equations, congruent, coordinates, ordered pair, median, function, inverse, standard deviation.

F. Knowledge of terms, symbolism and figures in synthetic plane and solid geometry.

G. Knowledge of terms and symbolism in elementary analytic geometry, trigonometry, and elementary probability and statistics.

H. Knowledge of properties of a field.

I. Knowledge of definitions of the trigonometric functions and their relations, functions of special angles.

J. Knowledge of basic geometric facts, such as the Pythagorean Theorem, angle relationships in circle and triangle, mensurational formulas.

K. Knowledge of laws of operation for exponents and logarithms.

L. Identification of graphs of circle, parabola, hyperbola.

M. Knowledge of simple probability and terms in probability.

N. Knowledge of symbols in logic, such as: $\cup, \cap, \subset, \therefore$.

O. Knowledge of business and commercial terms in common usage, such as: gross, net, profit, loss, selling price, cost, discount, successive

discount, prorate, compound interest, tax millage, exemption, debit, credit.

P. Knowledge of scientific units, such as: calorie, B.T.U., foot-pound, ohm, ampere, volt, watt, coulomb, erg, dyne, poundal, lumen, foot-candles, roentgen, angstrom, light-year, nail sizes, wire gauge, horsepower.

Q. Knowledge of scientific notation.

R. Knowledge of metric system.

S. Knowledge of necessary and sufficient conditions, converse, inverse, contrapositive, counterexample.

Adult

A. Facts of arithmetic, algebra, geometry, and trigonometry.

B. Units of measure and their relationships, English and metric.

C. Knowledge of measuring instruments.

D. Knowledge of commercial, social, and scientific terms.

E. Knowledge of statistical terms, such as mean, median, mode, correlation, standard deviation, frequency distribution, normal curve, skewing, peaked.

F. Use of computers.

II. *Perform mathematical manipulations.*

The second level of ability will be assessed by means of tasks which require the examinee to carry out single operations and procedures (or sequences of these), that have been previously learned and are specifically requested. Such tasks will require developed skill but will not require any decision as to which process or sequence of processes is needed (e.g., algorithm). It is in this category that all straightforward computation is included from simple addition to operations with complex numbers; it also includes solution of equations, evaluation of functions, etc. In any case the tasks which the examinee is required to perform involve only the rote application of learned techniques.

Age 9

A. Addition and subtraction including carrying, borrowing, and checking.

B. Multiplication and division, including division with remainders.

C. Finding the ordinal number of a page in book, or room in a corridor.

D. Addition and subtraction of fractions having the same denominator.

E. Changing fraction to higher or lower terms (equivalent fractions).

F. Adding and subtracting numbers involving mixed numerals.

G. Multiplication of fractional numbers (e.g., 1/3 X 1/2).

H. Reading measuring devices such as a ruler, thermometer, clock, weighing scale, calendar.

I. Reading picture and bar graphs.

J. Making a number line and a graph of the solution set of an equation.

K. Rounding to tens, hundreds, thousands.

Age 13

A. Arithmetic computations, approximate computation.

B. Manipulation of simple algebraic expressions.

C. Geometrical constructions: bisecting an angle, drawing a line perpendicular to a given line.

D. Making measurements with ruler, protractor, thermometer.

E. Finding distances on maps and scale drawings.

F. Using formulas by substituting values.

G. Reading all types of graphs.

H. Solving proportions.

I. Finding square roots, by table and by using an algorithm.

Age 17

A. Arithmetic computation; computation with approximate data.

B. Manipulation of algebraic expressions, including inequalities.

C. Geometrical constructions.

D. Computation with logarithms.

E. Slide rule computation.

F. Using a desk calculator.

G. Using conversion relations.

H. Using carpenter and engineering rules; transit, sextant, vernier, micrometer, meter sticks, time devices, and weighing instruments.

I. Evaluation of determinants; operations with matrices.

J. Solving simultaneous equations—linear and quadratic.

K. Using nomographs.

L. Application of tests of divisibility.

M. Finding square roots.

N. Interpolation and extrapolation with a table.

O. Finding relative error of a measurement.

P. Synthetic division.

Q. Expanding a binomial.

Adult

A. Arithmetic, algebraic, geometric, and trigonometric computations; approximate computation.

B. Using a desk calculator.

C. Using a wide variety of measuring instruments.

D. Making accurate scale drawings using ruler, compasses, and protractor.

E. Finding data in tables.

F. Keeping a check book.

III. *Understand mathematical concepts and processes.*

This third level will include tasks which demonstrate understanding of concepts and of mathematical processes through the ability to transform or translate from one type of "language" or symbolism to another. Since the demonstration of comprehension must involve communication, this level will assess comprehension through tasks which include the following possible kinds of translations or transformations within a mathematical context:

– verbal to mathematical (e.g., words to symbols)
– mathematical to verbal (e.g., symbolism to verbal)
– mathematical to mathematical (e.g., translating from one kind of representation to another like an equation to a graph of the equation)
– mathematical to physical (e.g., use of charts to explain fractions)
– physical to mathematical (e.g., developing formulae for physical phenomena)
– verbal to verbal (e.g., explanation)

Age 9

A. Telling and demonstrating the meaning of a number, such as using an abacus to show the meaning of 238.

B. Using sets of objectives to show what the addition of two numbers means; also to show the process as regrouping by tens and ones.

C. Demonstrating the meaning of multiplication and division by using the number line.

D. Showing with diagrams the meaning of a fraction and higher or lower terms.

E. Demonstrating addition, subtraction, multiplication, and division on the number line.

F. Exhibiting an understanding of measurement by selecting a new unit and measuring a familiar object with it (e.g., measuring the length of a room with a pupil's foot or forearm, and comparing this measurement with that obtained by using a standard unit).

G. Showing an understanding of odd and even numbers by locating a house on a street or by predicting the position of an odd page in a book.

H. Explaining the meaning of number frames such as $\square$ and Δ. Explaining under what conditions $\square$ and $\Delta = 10$.

I. Explaining the inverse of addition and subtraction and relating this to checking computation.

J. Using the idea of ratio in reading a simple floor plan or map.

K. Translating a verbal statement like "If a number is increased by 5, the result is 12" into a mathematical sentence such as "N + 5 = 12."

L. Showing why 3 x 4 equals 4 x 3.

M. Demonstrating the meaning of "less than" and "greater than."

Age 13

A. Translating a verbal statement into a mathematical sentence.

B. Representing a set of data with a graph.

C. Illustrating a geometric theorem by making sketches.

D. Translating a formula into an English statement.

E. Making a scale for measuring.

F. Writing a base ten numeral for a number in another base.

G. Describing a physical relationship with an algebraic equation. The relationship should fall within the 13-year-old's experience, such as a formula for prices.

H. Explaining what a linear relationship is.

Age 17

A. Making a graph of a function.

B. Finding the equation of a graph.

C. Making a slide rule with strips of graph paper.

D. Learning and using new notation, such as writing numerals in other bases.

E. Demonstrating the properties of a mathematical system such as rotations of a square, or a finite number system.

F. Using principles of logic in making a proof.

G. Explaining the long division algorithm in terms of successive subtractions.

H. Explaining what a quadratic fraction is; what an exponential function is.

I. Changing a problem in compound interest or installment buying into one of series summation.

J. Interpreting statistical data.

Adult

A. Explaining mathematical principles and operations.

B. Stating commercial problems in mathematical terms.

C. Quantifying industrial problems.

D. Interpreting statistical data; stating summaries and conclusions.

E. Understanding computer processes and uses.

F. Defining a concept operationally.

IV. *Solving mathematical problems–social, technical, and academic.*

Assessment of the ability to solve problems requires the examinee to demonstrate the ability to select knowledge, skills, information, and techniques which are needed to solve a particular problem and to apply such background in actually solving the problem.

Included in an assessment of the ability to solve problems will be tasks ranging from routine to unfamiliar, from specific to abstract, and from those whose solutions are straightforward to those which require ingenuity and insight.

Included under IV will be much of the consumer mathematics used by the majority of adults. Also included will be the ability to follow a proof, find a flaw in a proof, construct a deductive proof, as in a plane geometry problem.

The common characteristic of tasks in this category will be that they require the individual to analyze a problem and determine a sequence of steps which will lead to a clearly specified outcome (whether the outcome is finding the cost of a purchase or proving a theorem).

Age 9

A. Basic arithmetic reasoning in appropriate settings.

B. Estimating answers to computations and problems.

C. Making change.

D. Finding averages and rates of speed.

E. Finding areas of squares, rectangles, and triangles; finding volumes of cubes and other rectangular solids.

F. Finding perimeter of rectangle, triangle, and other polygons.

G. Determining the distance traveled from two odometer readings, the change in temperature from two thermometer readings; finding increments by taking differences.

H. Using proportions, as in recipes and mixing punch.

Age 13

A. Reasoning in problems involving arithmetic skills.

B. Solving simple algebra problems.

C. Stating generalizations about relation in geometry.

D. Estimation.

E. Making change.

F. Finding information from tables and graphs.

G. Computing areas that require making of measurements.

H. Making a scale drawing.

I. Conversion of units of measure.

J. Finding an unknown distance using similar triangles or a trigonometric ratio.

K. Calculating a distance from a map.

Age 17

A. Solution of triangles using trigonometric ratios.

B. Proving trigonometric identities.

C. Solving problems in algebra.

D. Proving geometric originals.

E. Interpreting tables and graphs.

F. Applying formulas.

G. Evaluation of determinants.

H. Computing with complex numbers.

I. Solving three dimensional locus problems.

J. Using Maxwell diagrams to solve problems involving forces.

K. Navigation problems using vectors.

L. Locating a flaw in geometric proof.

M. Locating a flaw in an algebraic proof.

N. Solving problems involving series and summation.

O. Solving problems in symbolic logic.

P. Solving surveying problems.

Q. Measuring, cutting, and folding patterns in cloth, paper, wood, metal.

R. Solving shop problems which involve inside and outside measurement, thread cutting, and mitering.

S. Solving bookkeeping and accounting problems.

T. Solving time zone problems.

Adult

A. Arithmetic, algebraic, and geometric exercises.

B. Solving problems in business arithmetic.

C. Solving machine shop mathematics problems.

D. Using vectors to solve problems in navigation and mechanics.

E. Comparing prices.

F. Reading blue prints and maps.

G. Computing taxes (e.g., real estate).

H. Computing investment returns.

I. Computing capital gains.

V. *Using mathematics and mathematical reasoning to analyze problem situations, define problems, formulate hypotheses, make decisions, and verify results.*

This level is a combination of those high level mathematical abilities which are open-ended and which require the use of mathematical techniques and patterns of thought in an independent and constructive way.

Tasks in this category include those which assess the ability to transfer and utilize knowledge in new situations, to recognize patterns, to draw conclusions from given data, to plan for the future on the basis of present information, and to use mathematical reasoning to make optimum decisions.

Tasks in this category also include the ability to recognize the existence of a problem, the ability to state it formally, the ability to formulate hypotheses, and the ability to ascertain if the problem has a unique solution. Assessment of the sufficiency of conditions and the determination of the minimum conditions necessary for proof, the disproof of hypotheses by counterexample, and proof by induction all come under this heading.

Age 9

A. Recognizing patterns and making simple generalizations involving number and geometrical relationship.

B. Adapting a geometric pattern to a limited area.

C. Comparative buying.

D. Stretching an allowance.

E. Planning a garden.

F. Planning a party.

G. Comparing populations using tables, graphs, averages, and other data.

Age 13

A. Recognizing patterns and making simple generalizations involving

number and geometrical relationship.

B. Consumer buying.

C. Budgeting.

D. Planning a trip.

E. Exploring number arrays.

F. Solving novel problems and puzzles.

G. Discovering geometric relationships by investigating a variety of geometric situations.

H. Drawing conclusions by gathering appropriate data, (e.g., school absences).

Age 17

A. Geometric experiments including drawing figures, making constructions, making measurements, folding paper, and making models in order to discover generalizations inductively.

B. Recognizing patterns and making generalizations about numerical and algebraic configurations.

C. Solving novel problems, puzzles, and recreations.

D. Comparative buying.

E. Planning personal finances.

F. Budgeting.

G. Cost estimating, such as planning a trip, remodeling a home.

H. Discovering fallacies in consumer advertising involving statistical data and graphs.

I. Detecting flaws in arguments, such as advertising and propaganda.

J. Assembly and presentation of statistical evidence in support of an argument.

K. Curve fitting.

L. Making a survey: defining the problem, designing questions, taking a

sample, collecting data, summarizing data, drawing conclusions.

Adult

A. Filling out complex income tax returns.

B. Recognizing patterns and making generalizations involving numerical, algebraic, and geometric data.

C. Solving novel problems requiring new approaches.

D. Planning personal investments.

E. Budgeting—business and personal.

F. Cost estimating for domestic or business purposes.

G. Detecting flaws in advertising or propaganda arguments.

H. Reading and interpreting business articles and reports.

I. Planning a construction job.

J. Buying on the basis of comparative studies.

K. Deciding when to make a major purchase (e.g., a new car) by considering depreciation, expenses, financing costs.

VI. *Appreciation and use of mathematics.*

A. Recognizing the importance and relevance of mathematics to the individual and to society.

From age 9 and up, there should be a recognition of the importance and relevance of mathematics to the individual and to society. This subobjective (VI.A.) does not necessarily involve enjoyment of mathematics or participation in the development of ideas, but rather it focuses on the acceptance of mathematics as being worthwhile—i.e., the individual recognizes that mathematics is necessary whether or not he uses it or enjoys studying it. For example, the individual should recognize the contribution that mathematics has made to the progress of civilization, especially in the sciences. There should also be appreciation of the elegance, economy, and techniques of mathematics. Of course, the level of sophistication of such appreciations should increase with age; nevertheless, some manifestation of these attitudes should appear at all age levels.

B. Enjoyment of mathematics.

In addition to having an appreciation for the importance of mathematics, the individual should also enjoy the subject and its specialized techniques (e.g., using compasses and working with numbers). Emphasis should be placed on the enjoyment involved in acquiring a knowledge of mathematics and in the satisfaction gained from using it rather than on the amount that is learned. The corollary of this is also important—i.e., the individual should not hate or fear mathematics. These attitudinal goals are especially important during the school years since they are likely to influence how much mathematics an individual is willing to study, and therefore, have at his disposal.

C. Using the content and techniques of mathematics.

When the mathematics is relevant and appropriate, individuals should use what they have learned. Because the amount of knowledge varies with age level, evidence of a willingness to use mathematics will take different forms at different age levels.

D. Participation in the learning of mathematics beyond that which is merely required, and actively seeking to further personal development in the area of mathematics.

The fourth subgoal relates to the individual's development of a curiosity about mathematics as well as a readiness to engage in activities in this area (i.e., independent of school and/or job assignments). In contrast to the objectives in other categories, independent action rather than reaction is stressed. This goal emphasizes that the individual should actively seek participation and further development of his skills in mathematics (as indicated by such things as reading about the "new" math and tackling strange looking problems). This is opposed to merely passing judgment or using the principles learned when this was required. It is expected that such interests will not develop before the age of 13; however, once developed, they will probably carry through into adult life.

Appendix

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Mrs. Gerald Chapman, Former School Board Member and State Legislator, Arlington Heights, Illinois

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National Citizens Committee for Support of Public Schools
National Conference of Christians and Jews
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Parochial Educational Organizations
State and Local Governmental Committees on Education
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